A note on the yaw attitude modeling of BeiDou IGSO-6

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Introduction

On March 29, 2016, China launched another second-generation BeiDou Navigation Satellite System (BDS) satellite, the 22nd in the country's GNSS program. Designated BDS IGSO-6, the spacecraft lifted off from the Xichang Satellite Launch Center atop a Long March 3A carrier rocket to enter a slot in inclined geosynchronous orbit (IGSO). Based on the Dong Fang Hong 3A (DFH-3A) bus, a medium-capacity platform designed by the China Academy of Space Technology (CAST), the satellite features a phased array antenna for L-band navigation signals and a laser retroreflector array (LRA) on its Earth-facing +z panel (Figure 1). It began transmitting navigation signals on April 6, 2016, according to tracking data from the International GNSS Service (IGS) receiver network.

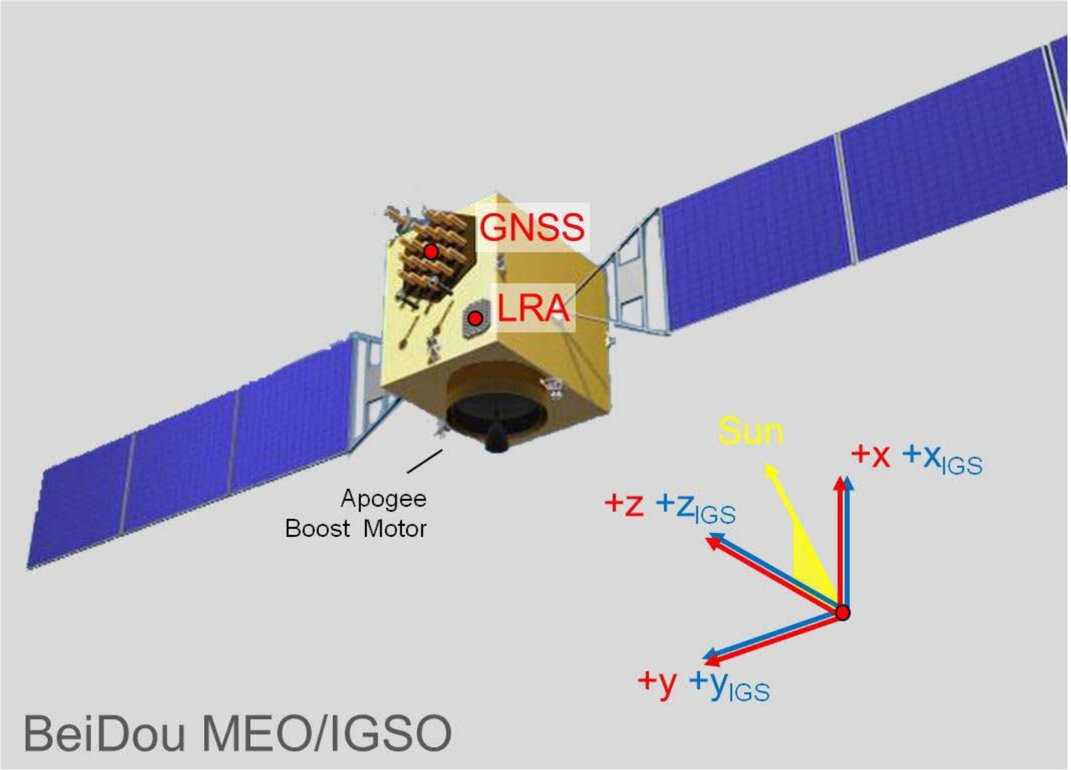

Figure 1: Artist's view of BeiDou Medium Earth Orbit (MEO) / Inclined Geosynchronous Orbit (IGSO) spacecraft [1].

Surprisingly, although the platform design has remained unchanged, IGSO-6 seems to step out of line and behaves differently in terms of its attitude than other second-generation BDS IGSOs and MEOs [2]. Since the L-band transmitter's antenna phase center position is offset from the spacecraft's z-axis by 0.55 m, one can use the reverse point positioning (RPP) technique to estimate the satellite's yaw state on an epoch-by-epoch basis with an accuracy of a few degrees. The results from such RPP analysis provide strong evidence that, when the Sun gets close to the satellite’s orbital plane, IGSO-6 does not enter orbit normal (ON) mode, as it used to know it for this type of spacecraft. Rather than holding its yaw attitude fixed when the acute angle (“beta prime”) between the solar vector and orbit plane falls below 4 deg, the satellite keeps rotating, or “yawing,” around the Earth-pointing z-axis, similar to a Galileo satellite. In other words, IGSO-6 performs yaw steering (YS) to maintain nominal yaw orientation at all times, even during the Earth’s shadow crossing. To keep its +x side facing the Sun, the satellite performs noon and midnight turn maneuvers, rotating the spacecraft up to 180 deg around the z-axis every half orbit (~12 hours). The nominal theoretical rotation rate for these yaw turns increases as the beta prime angle approaches zero. Before the nominal rate is about to exceed the satellite’s maximum hardware rate, the attitude control system transitions from standard YS to smoothed YS. In this way, the yaw rate remains always within the physical limits. A mathematical description of how to model the satellite’s yaw motion is given below.

BeiDou yaw steering model

A simple attitude model for IGSO-6 has been developed by empirically fitting a special inverse tangent function to a series of RPP yaw estimates. The model will be subsequently referred to as the BeiDou YS model. It guarantees seamless transition from one attitude mode to the other, that is, from standard YS to smoothed YS and vice versa. The key parameter that governs the attitude mode is the Sun’s elevation angle ß. When ß is smaller than –$ß_0$ or greater than +$ß_0$, the model assumes the satellite is performing standard YS. If ß falls between –$ß_0$ and +$ß_0$, the smoothed YS law takes over. The value for the critical Sun elevation angle $ß_0$ was found to be 2.8 deg.

The idea behind this algorithm goes back to [3]. We start out with the well-known formula for the ideal or nominal yaw angle

$$\psi_n = \mathrm{ATAN2}(-\tan\beta\,, \sin\eta), \tag{1}$$

where η is the geocentric orbit angle between the satellite and orbit midnight, measured in the direction of the spacecraft’s motion, and ATAN2 the usual FORTRAN function of $\tan^{-1}$, giving unambiguous results in the range of ±180 deg.

To keep the change rate of the yaw angle at orbit noon and orbit midnight from becoming infinitely large, we now replace the actual Sun elevation angle ß in equation (1) using the following modified Sun elevation angle

$$\beta_d = \beta + f \cdot (\mathrm{SIGN}(\beta_0, \beta) - \beta), \tag{2}$$

where SIGN($ß_0$, ß) is a FORTRAN function returning the value of $ß_0$ with the sign of ß. This modified Sun elevation angle ensures a minimum angular distance of $ß_0$ between the Sun’s vector and the spacecraft’s z-axis. In this way, the yaw rate remains continuously below a certain maximum value. We will come back to the yaw rate in a little bit.

The formula for the yaw angle now becomes

$$\psi = \mathrm{ATAN2}(-\tan\beta_d\,, \sin\eta). \tag{3}$$

Note that the *f* in equation (2) is a bell-shaped smoothing function of the orbit angle η. As the name implies, the function is designed for "smoothing" the yaw angle around the critical orbit positions (η = 0, 180 deg). After some trial and error, we arrived at the following expression for the smoothing function:

$$f = \begin{cases} \dfrac{1}{1 + d \cdot \sin^4\eta} & \text{for } \beta_0 \le |\beta| \\ 0 & \text{for } \beta_0 > |\beta| \end{cases}, \tag{4}$$

where d = 80000 is a dimensionless constant.

For the yaw rate, we need to first find the partial derivative of the yaw angle with respect to the orbit angle η. In order to keep things simple, we ignore the time variation of the slow-changing Sun elevation angle β. Straightforward algebra then yields

$$\dot{\psi} = \frac{\mathrm{d}\psi}{\mathrm{d}\eta} = \frac{\cos\eta \cdot \tan\beta_d - \dot{\beta}_d \cdot \sin\eta \cdot \sec^2\beta_d}{\tan^2\beta_d + \sin^2\eta} \tag{5}$$

with

$$\dot{\beta}_d = \dot{f} \cdot (\mathrm{SIGN}(\beta_0, \beta) - \beta) \tag{6}$$

and

$$\dot{f} = \begin{cases} -4 \cdot d \cdot f^2 \cdot \cos\eta \cdot \sin^3\eta & \text{for } \beta_0 \le |\beta| \\ 0 & \text{for } \beta_0 > |\beta| \end{cases}. \tag{7}$$

The yaw rate in units of degree per second is

$$\frac{\mathrm{d}\psi}{\mathrm{d}t} = \dot{\psi} \cdot \dot{\eta}, \tag{8}$$

where $\dot{\eta}$ = 0.004178 deg/s is the average orbital velocity of a BeiDou IGSO satellite. The maximum yaw rate is

$$\left.\frac{\mathrm{d}\psi}{\mathrm{d}t}\right|_{\beta=\eta=0} = \frac{\dot{\eta}}{\tan\beta_0} \approx 0.085\ \mathrm{deg/s}. \tag{9}$$

For a BeiDou MEO satellite that flies almost twice as fast with $\dot{\eta}$ = 0.007762 deg/s, the maximum yaw rate value would be 0.159 deg/s.

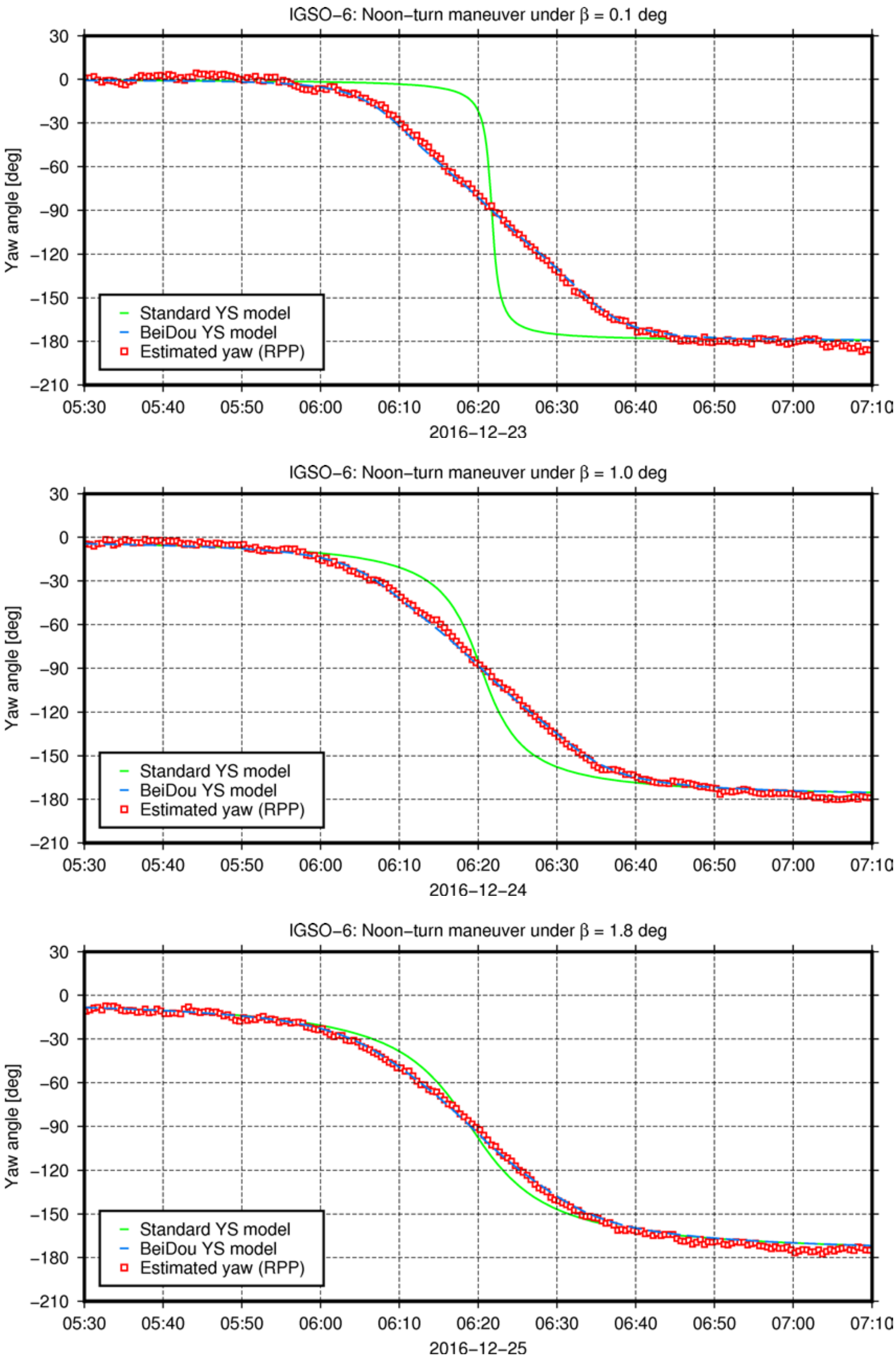

Figure 2: Yaw angle of IGSO-6 during noon turn maneuvers from December 23–25, 2016.

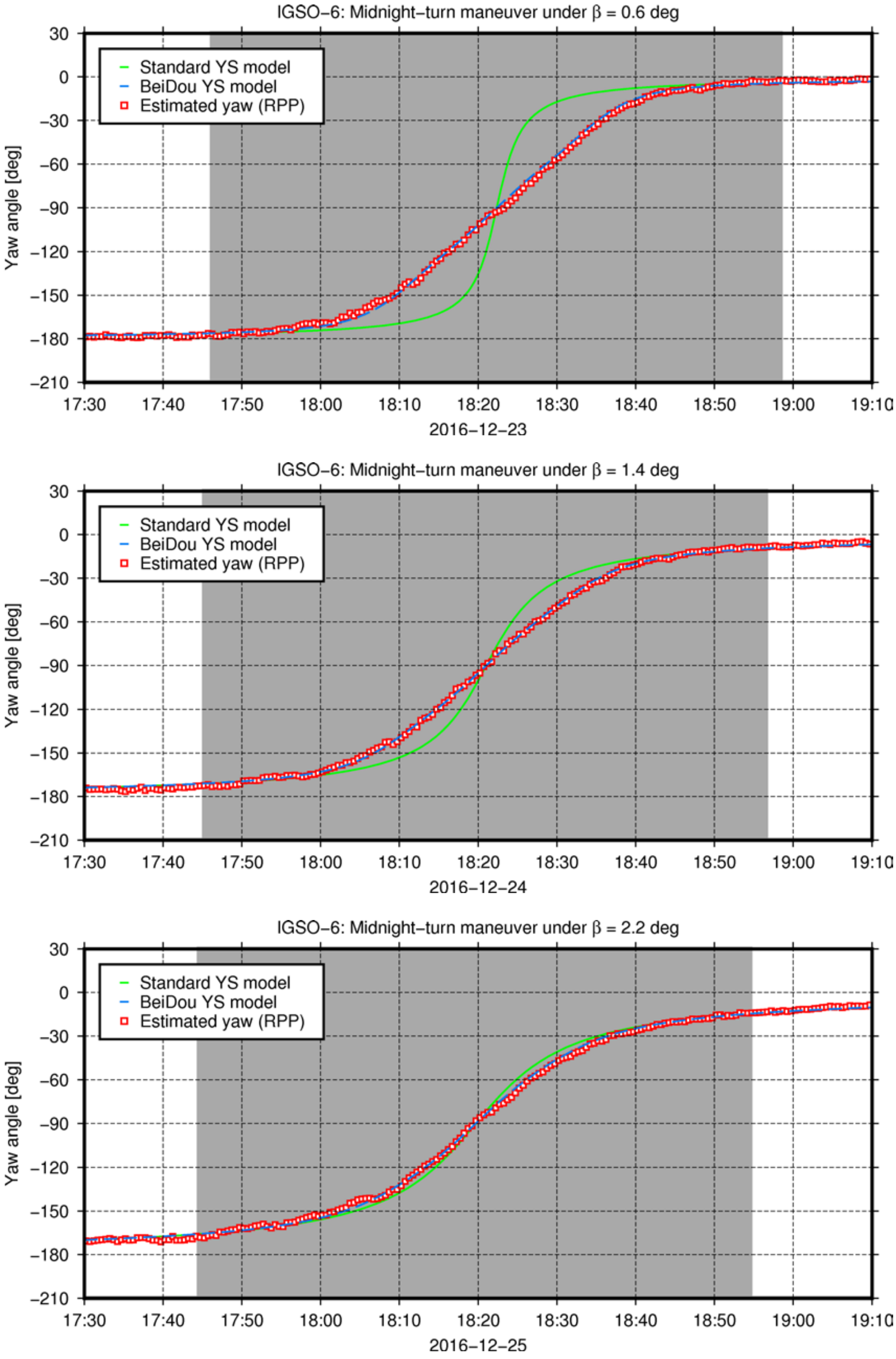

Figure 3: Yaw angle of IGSO-6 during midnight turn maneuvers from December 23–25, 2016.

Figure 2 and Figure 3 show examples of noon and midnight turn maneuvers of IGSO-6 under a very low beta angle. The plots display the yaw angle estimates from the RPP alongside the yaw angle values from the standard YS model of equation (1) and the new BeiDou YS model of equation (3). They provide a rough idea of how well the new model fits the “true” yaw angle.

Figure 4 shows the RPP estimates after removal of the standard YS (green) and the BeiDou YS (blue) model values. The plot displays data from three eclipse seasons (June 2016, December 2016, and June 2017). It demonstrates the new model’s ability to predict the yaw angle with the same accuracy (~3 deg RMS) under a low beta angle as it does under a higher beta angle. With the standard YS model, the predicted yaw angle under low beta angle would be in error by up to 90 deg.

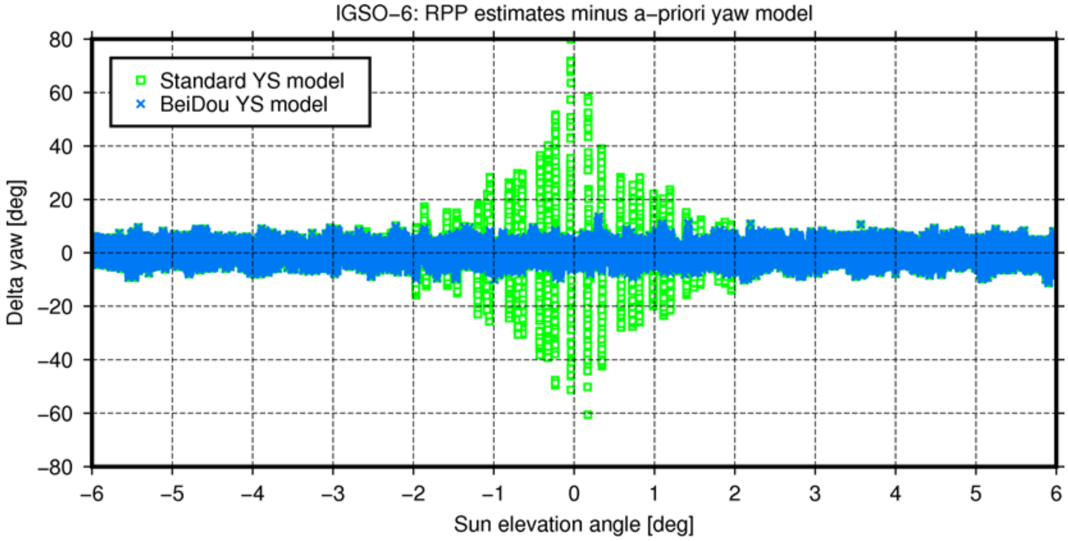

Figure 4: RPP estimates with respect to the standard YS model (*green*) and the new BeiDou YS model (*blue*).

BeiDou YS model vs. YS-ON model

Employing the conventional YS-ON attitude model for IGSO-6 is inadvisable, as this model can introduce a measurable error in the satellite’s antenna phase wind-up correction. Since the wind-up error due to yaw mismodeling is common to all stations observing the spacecraft, the satellite clock parameter will inevitably “soak up” most of the effect. Figure 5 illustrates the impact on the satellite clock estimates as a function of ß when using the standard BeiDou YS-ON model (green) and the new BeiDou YS model (blue).

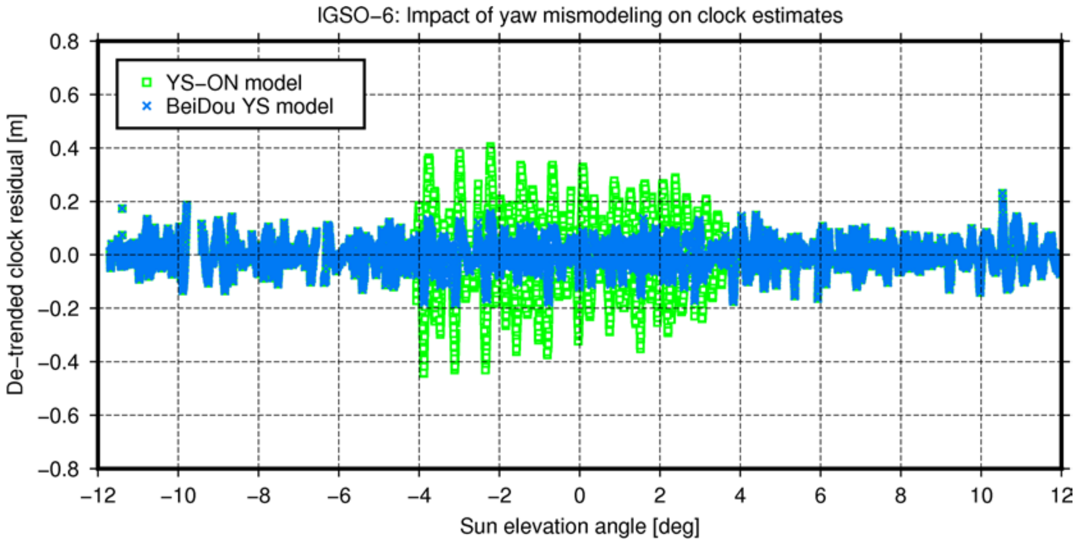

Figure 5: Satellite clock estimates after removal of a third-order polynomial when using the standard BeiDou YS-ON model (*green*) and the new BeiDou YS model (*blue*).

Failure to account for the right yaw attitude does not only affect the reconstitution of the radiometric ranges but also the modeling of the satellite orbit dynamics and of the LRA orientation. Since the center of the LRA is offset from the spacecraft's yaw axis by approximately 0.7 m, the total effect on the satellite laser ranging (SLR) residuals can easily reach the order of a few decimeters. Figure 6 shows the SLR residuals when using the standard BeiDou YS-ON model (green) and the new BeiDou YS model (blue). The consequences of taking the wrong yaw model on the day-to-day orbit overlap residuals can be seen in Figure 7.

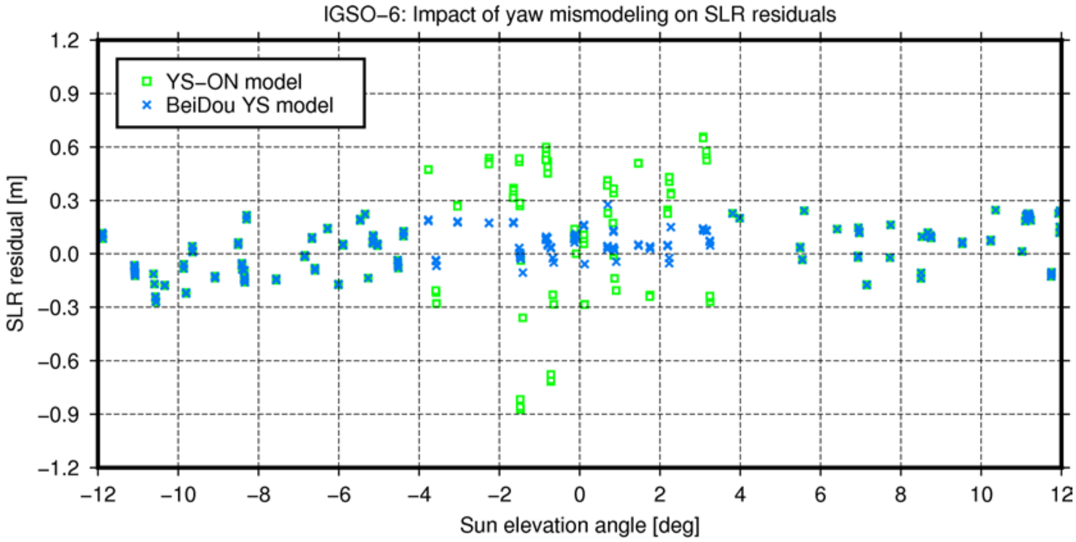

Figure 6: One-way SLR residuals when using the standard BeiDou YS-ON model (*green*) and the new BeiDou YS model (*blue*).

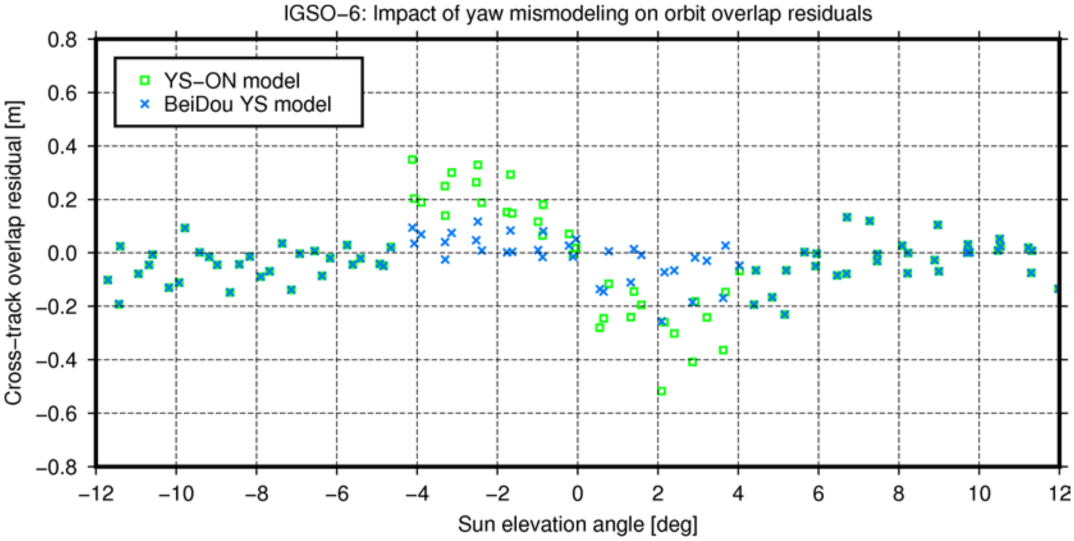

Figure 7: Orbit overlap residuals in cross-track direction when using the standard BeiDou YS-ON model (*green*) and the new BeiDou YS model (*blue*).

Summary

The development and performance of a simple yaw attitude model for the BDS IGSO-6 satellite has been described. The model fits the yaw angle estimates from RPP with approximately 3 deg of accuracy (RMS).

Outlook

At the time of this writing (November 2017), two more second-generation BeiDou spacecraft, MEO-6 (C015) and IGSO-1 (C005), were found to have abandoned their ON mode in favor of continuous YS. Both satellites now precisely follow the YS law described in this technical note (Figure 8). The last time the two vehicles entered ON mode was in October 2016 (MEO-6) and in March 2017 (IGSO-1).

Since the attitude law is apparently not “hardwired” into the attitude control circuit but is reconfigurable by the spacecraft operators, we might expect further BeiDou-2 satellites in the near future to alternate from the YS-ON mode to continuous YS.

Rumors are swirling that the third generation of the BeiDou MEO and IGSO spacecraft also no longer employs the ON mode. Whether these new BeiDou-3 satellites follow the same YS algorithm as the group of the continuous yawing BeiDou-2 satellites remains to be seen.

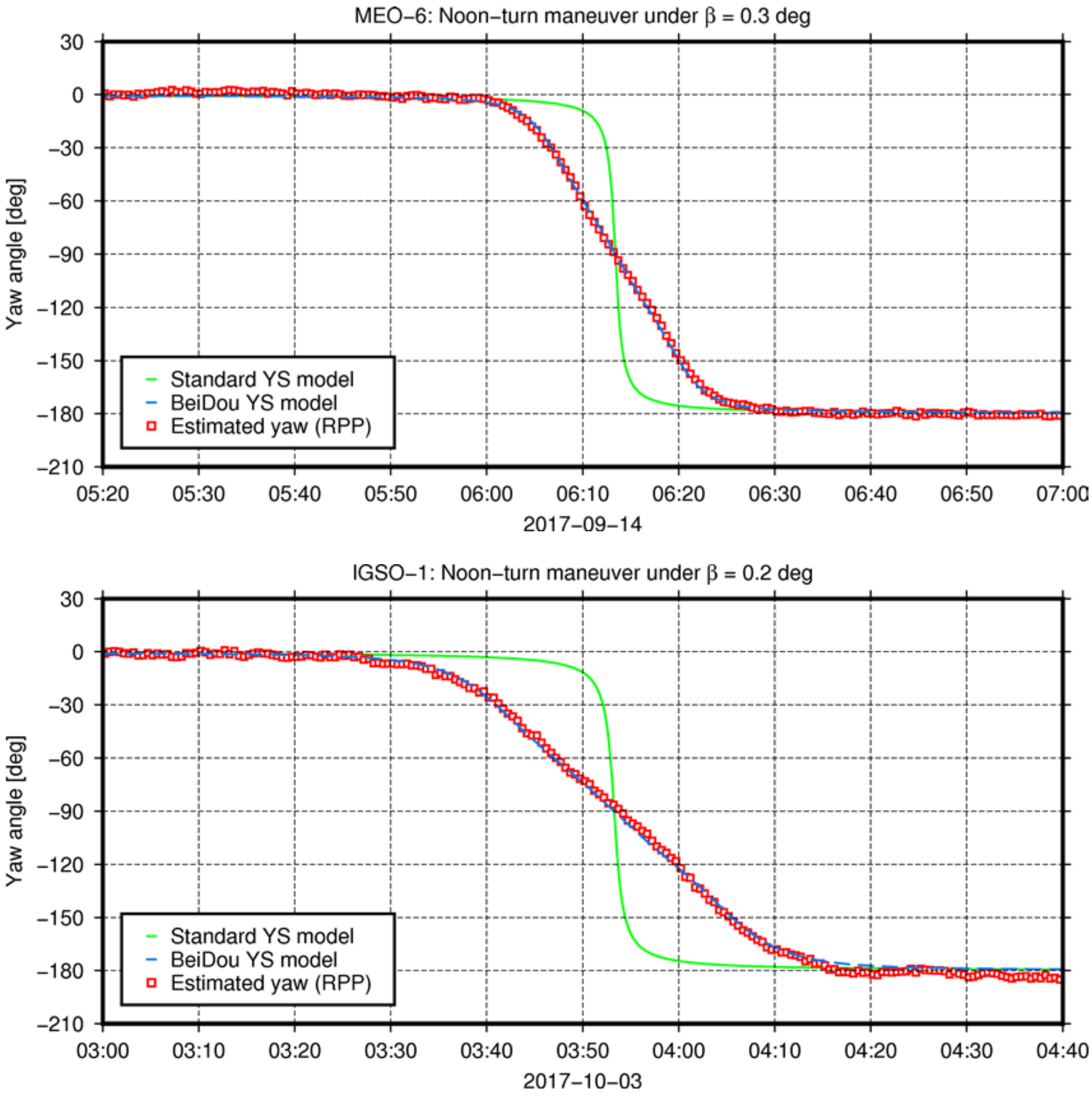

Figure 8: Yaw angle of MEO-6 (*top*) and IGSO-1 (*bottom*) during noon turn maneuver.

References

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[3] Osterlin W., Ebert, K. (2003): Dynamic Yaw Steering Method for Spacecraft. European Patent No. 03024205.